
Nozzle Thrust Increase by Rotation of the Flow

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Until recently, in the determination of the optimal supersonic axisymmetrical nozzles the gas motion was considered without rotation around the axis. The problem for equilibrium gas flow is studied completely enough⁽¹⁻⁶⁾. The rejection of the rotation of the flow is the restriction that can decrease the nozzle thrust, other conditions being equal. It is easy to show that the uniform flow parallel to the axis supplies maximum thrust if the given length of a nozzle is zero or allows the uniform flow. If the given length does not allow that then the use of the arbitrariness of gas rotation may increase the nozzle thrust. A simple example shows that such a possibility takes place.

SYMBOLS

- a sound velocity
- b parameter defined by eqn. (4)
- h parameter defined by eqn. (9)
- p pressure
- q parameter defined by eqn. (9)
- Q mass flow through tube
- r co-ordinate
- T thrust
- u, v velocity vector components
- w peripheral velocity
- x Cartesian co-ordinate
- X length of nozzle
- z parameter defined by eqn. (9)

- γ circulation

- θ angle defined by eqn. (9)
 λ_1, λ_2 constants defined by eqn. (12)
 μ angle defined by eqn. (9)
 ρ density
 χ isentropic exponent
 ψ stream function

Axisymmetrical isoenergetical isentropical motion of the perfect gas is described as follows

$$\left. \begin{aligned}
 \frac{\partial r \rho u}{\partial x} + \frac{\partial r \rho v}{\partial r} &= 0, & \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r} &= 0 \\
 \rho &= a^{2/(x-1)}, & a^2 &= \frac{\chi-1}{2} \left[\frac{\chi+1}{\chi-1} - u^2 - v^2 - \frac{\gamma^2(\psi)}{r^2} \right] \\
 d\psi &= r\rho(u dr - v dx)
 \end{aligned} \right\} \quad (1)$$

where

- x, r are Cartesian co-ordinates of meridional plan of the flow,
 u, v are the velocity vector components,
 ρ is the density,
 a is the sound velocity,
 γ is the circulation,
 ψ is the stream function and
 χ is an isentropic exponent.

All the variables are dimensionless.

The pressure p and the peripheral velocity w are determined by the equalities

$$p = \frac{1}{\chi} a^{2x/(x-1)}, \quad w = \frac{\gamma(\psi)}{r} \quad (2)$$

The second equation of system (1) can be replaced by

$$\frac{\partial}{\partial x} r(p + \rho u^2) + \frac{\partial}{\partial r} r \rho u v = 0 \quad (3)$$

Along characteristics of eqns. (1) the following relations take place

$$dr = \frac{v^2 - a^2}{uv \pm ab} dx, \quad b = \sqrt{(u^2 + v^2 - a^2)} \quad (4)$$

$$(av \pm bu) du + (-au \pm bv) \left(dv + \frac{a^2 r^2 + \gamma^2 v}{a^2 r^2 - \gamma^2 r} dr \right) = 0 \quad (5)$$

The upper and lower signs belong to different families of characteristics.

We consider the simple problem. Let the gas flow (Fig. 1) go into the nozzle with the contour ab through cylindrical tube with the contour sa . Let then the length of the nozzle X , the radius Oa and the mass flow be given. The complete enthalpy of the gas is constant. Among all the possible contours of the nozzle it is necessary to find the one which provides the maximum thrust.

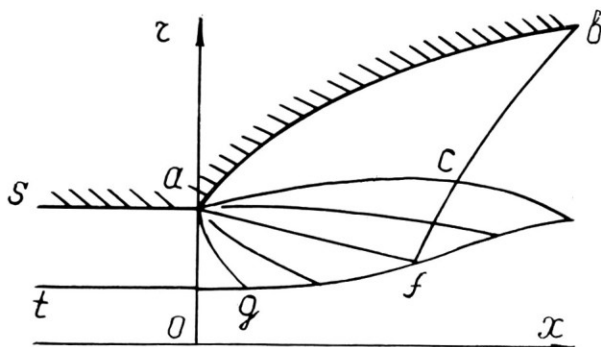


FIG. 1

We consider value γ to be constant. In this case the flow takes place above the line tgf . Below the line there appears a stagnation region with constant pressure.

If the flow in a tube does not depend on x then

$$u = u_0 = \text{const}, \quad v = 0 \quad (6)$$

We denote the pressure in the stagnation region and in the outer flow by p_∞ . The mass flow Q through the tube is determined by the integration of the last equation of (1) with respect to (6)

$$\frac{Q}{2\pi} = \left(\frac{\chi-1}{2}\right)^{1/(\chi-1)} u_0 \int_{r_0}^1 \left(\frac{\chi+1}{\chi-1} - u_0^2 - \frac{\gamma^2}{r^2}\right)^{1/(\chi-1)} r dr \quad (7)$$

$$r_0 = \gamma \left[\frac{\chi+1}{\chi-1} - u_0^2 - \frac{2}{\chi-1} (\chi p_\infty)^{(\chi-1)/\chi} \right]^{-1/2}$$

where r_0 corresponds to $p = p_\infty$ and is determined by the first equation of (2). Equations (7) allow u_0 and r_0 to be found if Q and p_∞ are given. The characteristic of the second family ag can be found by means of equalities (4) and (6).

The full thrust T is defined by the equality

$$\frac{T}{2\pi} = \int_{r_0}^1 \left[\frac{\chi+1}{2\chi} (1+u_0^2) - \frac{(\chi-1)\gamma^2}{2\chi r^2} \right] \rho r dr + \int_1^{r_b} \rho r dr - \frac{p_\infty}{2} (r_b^2 - r_0^2) \quad (8)$$

We consider the region $agfba$ and integrate eqn. (3). According to the Green formula we transform the integral to a contour one. Using the relations along a stream line and characteristics we have

$$\left. \begin{aligned} \frac{T}{2\pi} &= \int_{r_f}^{r_b} q(\mu, r) z(\mu, r) \left[\frac{\sin \mu}{\chi} + \frac{\cos \theta}{\sin(\theta + \mu)} \right] r dr + \frac{p_\infty}{2} (r_f^2 - r_b^2) \\ \mu &= \arcsin \frac{a}{\sqrt{(u^2 + v^2)}}, \quad \theta = \arctg \frac{v}{u}, \quad q = \left[\frac{(\chi - 1)h}{\chi - \cos 2\mu} \right]^{1/2} \\ z &= \left(\frac{\chi - 1}{2} h \frac{1 - \cos 2\mu}{\chi - \cos 2\mu} \right)^{\frac{1}{2}[(\chi + 1)/(\chi - 1)]}, \quad h = \frac{\chi + 1}{\chi - 1} - \frac{\gamma^2}{r^2} \end{aligned} \right\} \quad (9)$$

In the same way the first equation of (1) gives

$$\frac{Q}{2\pi} = \int_{r_f}^{r_b} \frac{\rho q \sin \mu}{\sin(\theta + \mu)} r dr \quad (10)$$

The length of the nozzle X is

$$X = \int_{r_0}^1 \left\{ \frac{(\chi + 1)(u_0^2 - 1)r^2 + (\chi - 1)\gamma^2}{[\chi + 1 - (\chi - 1)u_0^2]r^2 - (\chi - 1)\gamma^2} \right\}^{1/2} dr + x_f - x_g - \int_{r_f}^{r_b} \text{ctg}(\theta + \mu) dr \quad (11)$$

We formulate the variational problem when γ is given. It is necessary to find functions $\mu(r)$ and $\theta(r)$ along characteristic fb which gives maximum to functional (9) under the isoperimetrical conditions (10), (11) under the differential condition (5) along the characteristic of the first family and with the given initial characteristic ag defined by equalities (6) and (7).

This problem is analogous to that on the definition of the optimal nozzle contour at $\gamma=0$. It can be solved by the same method⁽⁷⁾. Below are the results of the solution.

Contour sab has the discontinuity of the first derivative at point a . The flow in region $acfga$ is determined by eqns. (1), the given characteristic ag and condition (5) for characteristics of the first family at point a .

Functions $\mu(r)$ and $\theta(r)$ along characteristic cb are determined by equalities

$$zq \frac{\sin^2 \theta}{\cos \mu} = \lambda_1, \quad q \frac{\cos(\theta - \mu)}{\cos \mu} = \lambda_2 \quad (12)$$

where λ_1 and λ_2 are constant. Their magnitudes are calculated according to formulas (12) at point c so that μ and θ at this point are continuous.

Function $x(r)$ along cb is calculated by integration of eqn. (4).

Co-ordinates x_c and r_c and value r_b must satisfy equalities (10) and (11)

at given Q and X and equality

$$qz \left(\frac{\sin \mu}{\chi} - \frac{\sin \theta \cos \theta}{\cos \mu} \right) = p_{\infty}$$

at $x = x_b$. The last equality is the generalisation of Busemann condition⁽¹⁾ for rotating flows.

The general solution of similar problem by the method of Ref. 5 would allow us to determine function $\gamma(\psi)$. But the aim of the present paper is to show that the rotation of the flow allows the nozzle to increase thrust. Therefore the circulation was adopted to be constant. The calculations have been performed at $\chi = 1.4$ with counterpressure $p_{\infty} = 0.0002259$. The value of mass flow corresponds to $u_0 = 1.5$ when $\gamma = 0$. The lengths of nozzles X and values γ have been varied. The circulations showed that in this case the nozzles with rotating flow had 0.4 per cent stronger thrust than the one with $\gamma = 0$.

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DISCUSSION

Dr. K. Kraemer (AVA, Bunsenstr. 10, 34 Göttingen, W. Germany): Is the stagnant air region near the nozzle-axis bounded by a Helmholtz-discontinuity-surface of constant pressure? What is the physical meaning?

Yu. Shmyglevsky: The calculations refer to a non-viscous gas.